

# Image restoration for a rectangular poor-pixels detector

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## ABSTRACT

This paper presents a unique two-stage image restoration framework especially for further application of a novel rectangular poor-pixels detector, which, with properties of miniature size, light weight and low power consumption, has great value in the micro vision system. To meet the demand of fast processing, only a few measured images shifted up to subpixel level are needed to join the fusion operation, fewer than those required in traditional approaches. By maximum likelihood estimation with a least squares method, a preliminary restored image is linearly interpolated. After noise removal via Canny operator based level set evolution, the final high-quality restored image is achieved. Experimental results demonstrate effectiveness of the proposed framework. It is a sensible step towards subsequent image understanding and object identification.

**Keywords:** Image restoration, maximum likelihood, level set, Canny operator, poor-pixels detector

## 1. INTRODUCTION

Poor-pixels detector, with extremely small quantity of photosensitive cells (pixels) on its imaging surface, is designed for an embedded computer vision system. It is valuable in many applications such as micro unmanned aerial vehicles and micro biomimetic robots. As an example, a rectangular 25-pixels detector is shown in Figure 1. It has properties of miniature size, light weight and low power consumption, which are helpful in a highly-integrated battery-using device. Meanwhile, the image acquisition speed can be dramatically accelerated due to only a few photoelectric signals being processed [1].

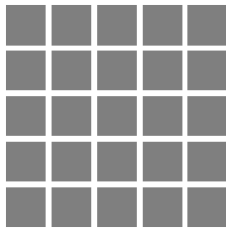


Figure 1: Pixel distribution of a rectangular 25-pixels detector (grey colour areas depict photosensitive cells)

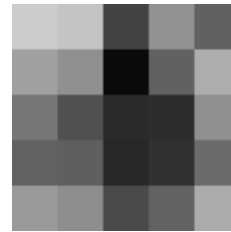


Figure 2: A synthetic extremely low-resolution rectangular image

An image captured by the rectangular 25-pixels detector, shown in Figure 2, has following features. (1) It is an extremely low-resolution image, like a mosaic image. (2) Grey information represented by image pixels is not with smooth continuity. Hence the contexture is hardly to be established. These make the image do not reflect the same or similar shape information as an object has in the real world. It is therefore difficult to identify an object and understand a scene based on such a single image.

To ensure further application of the rectangular poor-pixels detector in real problems, this paper presents a unique image restoration framework from a series of extremely low-resolution rectangular images shifted up to subpixel level. Different from traditional approaches in which the amount of information contained in the original low-resolution images should be larger than the amount of information required in the restored image [2-4], the designed framework only needs a few measured images to join the fusion operation. This is a guarantee to meet the demand of fast processing. However,

cutting down the data means losing the information. To be still able to achieve a high-quality restored image, a two-stage image restoration strategy is adopted in the proposed framework. Firstly, a preliminary restored image is linearly interpolated by maximum likelihood estimation. Secondly, as further restoration, Canny operator based level set evolution is applied for noise removal.

The rest of the paper is organized as follows. Section 2 introduces details of the proposed framework. Experimental results on synthetic images are presented and discussed in Section 3. Section 4 concludes the work with identified achievements.

## 2. METHODOLOGY

### 1.1 Mathematical model

To simply and efficiently model the problem,  $N$  extremely low-resolution images  $Y_k$  ( $k=1,2,\dots,N$ ) are acquired by shifting a rectangular 25-pixels detector up to subpixel level. As shown in Figure 3, they are different representations of an ideal high-resolution image  $X$ . The popular formulation is defined as [4]:

$$Y_k = D_k H_k F_k X + V_k \quad (k=1,2,\dots,N) \quad (1)$$

$F_k$  stands for the geometric warp operation that exists between  $X$  and an interpolated version of  $Y_k$  (interpolation is required in order to treat the image  $Y_k$  in the higher resolution grid).  $H_k$  is the blur matrix, representing the detector's PSF.  $D_k$  denotes the decimation process, representing the reduction of the number of observed pixels in the acquired images.  $V_k$  is Gaussian additive measurement noise with zero mean and auto-correlation matrix  $W_k = \sigma^2 I$ .

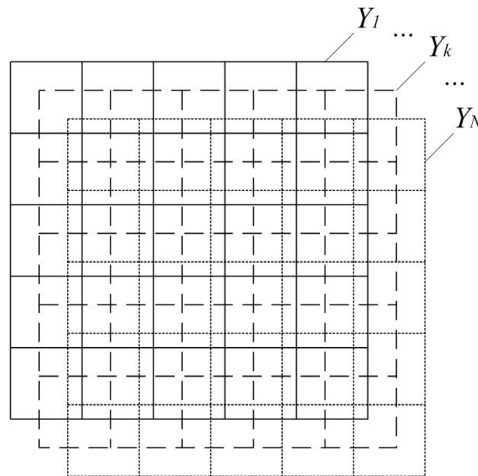


Figure 3: Modeling the image restoration problem based on  $N$  extremely low-resolution rectangular images

Traditionally, if the number of pixels in the measured image  $Y_k$  is  $M_k \times M_k$  and in the ideal image  $X$  is  $L \times L$ , one intuitive rule is:

$$L^2 < M_1^2 + \dots + M_k^2 + \dots + M_N^2$$

which can be explained as a requirement that the amount of information contained in the measured low-resolution images should be larger than the amount of information required in the restored high-resolution image. To estimate  $X$  well, plenty of  $Y_k$  are needed, which, however, may increase the computation and slow the operation. In this paper, to meet the demand of fast processing, fewer measured images are taken into the restoration. By shifting the rectangular poor-pixels detector only along the horizontal, vertical and diagonal directions, a special series of  $Y_k$  are obtained. This acquisition process is easy to realize but loses some useful information which needs to be compensated in the following operations.

## 1.2 Linear interpolation

The maximum likelihood estimation is employed to estimate  $X$  by maximizing the conditional probability density function  $P\{Y|X\}$ .

$$\hat{X}_{ML} = \underset{X}{\text{ArgMax}} P\{Y|X\} \quad (2)$$

Where  $\hat{X}_{ML}$  denotes the maximum likelihood estimate of  $X$  and  $Y$  denotes the group of  $Y_k$ . Based on the assumption that Gaussian additive noise vectors are mutually independent, Eq. (2) is done through the following least squares expression:

$$\hat{X}_{ML} = \underset{X}{\text{ArgMin}} \left\{ \sum_{k=1}^N [Y_k - D_k H_k F_k X]^T W_k^{-1} [Y_k - D_k H_k F_k X] \right\} \quad (3)$$

Differentiating Eq. (3) with respect to  $X$  and letting it be zero gives the well-known classic pseudo-inverse result:

$$P \hat{X}_{ML} = Q \quad (4)$$

where

$$P = \sum_{k=1}^N F_k^T H_k^T D_k^T W_k^{-1} D_k H_k F_k \quad (5)$$

$$Q = \sum_{k=1}^N F_k^T H_k^T D_k^T W_k^{-1} Y_k \quad (6)$$

For the purpose of image smoothness, locally adaptive regularization is adapted in Eq. (3) by using the Laplacian operator  $S$  and a weighting matrix  $V$ . Thus, Eq. (3) becomes to

$$\hat{X}_{ML} = \underset{X}{\text{ArgMin}} \left\{ \sum_{k=1}^N [Y_k - D_k H_k F_k X]^T W_k^{-1} [Y_k - D_k H_k F_k X] + \beta [SX]^T V [SX] \right\} \quad (7)$$

where  $\beta$  is a relaxing factor. Differentiating Eq. (7) again with respect to  $X$  and letting it be zero yields the same equation as Eq. (4) with a new term,  $\beta S^T V S$ , added to the matrix  $P$  as

$$P = \sum_{k=1}^N F_k^T H_k^T D_k^T W_k^{-1} D_k H_k F_k + \beta S^T V S \quad (8)$$

To solve the above equations, the conjugate gradient algorithm is chosen due to its relatively fast convergence and high quality result [5].

Although the above linear interpolation method is easy to operate and workable as well, there are still a lot of noises in the preliminary restored image. Additionally, due to the wanting information at the acquisition stage, the contour of the restored object is not clear either. To make further improvement of the restoration, Canny operator based level set evolution is then applied.

## 1.3 Noise removal

Among all filtering techniques, level set method [6-9], which is less sensitive to natural noise and more contrast preserving, has become popular due to its flexibility and capability in modeling complex structures. While Canny operator [10], which is robust to noise, is probably the most widely used edge detector. Unless the preconditions are particularly suitable, it is hard to find an edge detector that performs significantly better than the Canny operator. To combine their advantages, a modified level set method with Canny operator is presented.

Level set method is based on the partial differential equation. Its main idea is that the moving interface is looked as a zero level set in one higher dimensional space. The evolution equation of the level set function  $\varphi$  can be written as [6]:

$$\frac{\partial \varphi}{\partial t} + F \cdot |\nabla \varphi| = 0 \quad (9)$$

where  $F$  is a speed function determining the diffusion of the moving interface opposite to its normal direction. For the problem of image noise removal,  $\varphi$  is replaced by the image data  $I_m$ , and  $F$  is usually a kind of curvature flow which depends on  $I_m$ . Thus, Eq. (9) is rewritten as:

$$\frac{\partial I_m}{\partial t} + F \cdot |\nabla I_m| = 0 \quad (10)$$

According to the definition of Canny operator [10],  $\mathbf{n}$ , the normal to the direction of a detected edge, can be estimated well from the smoothed gradient direction

$$\mathbf{n} = \frac{\nabla(G * I_m)}{|\nabla(G * I_m)|} \quad (11)$$

where  $G$  is the two dimensional Gaussian function. At such an edge point, the edge strength  $E$  can be calculated by

$$E = \sqrt{E_x^2 + E_y^2} \quad (12)$$

where  $E_x = \frac{\partial G}{\partial x} * I_m$  and  $E_y = \frac{\partial G}{\partial y} * I_m$ .

Replacing  $\nabla I_m$  in Eq. (10) by  $\mathbf{n}$ , the normal direction to an edge, and selecting  $F$  in Eq. (10) with consideration of the edge strength  $E$ , the level set evolution equation with Canny operator is obtained as:

$$\frac{\partial I_m}{\partial t} + F_c \cdot \frac{|\nabla(G * I_m)|}{|\nabla(G * I_m)|} = 0 \quad (13)$$

$$F_c = \begin{cases} \text{mean flow} & \max(E) < T \\ \text{min/max flow} & \text{otherwise} \end{cases} \quad (14)$$

where  $\max(E)$  is the maximum of edge strength in  $3 \times 3$  neighborhoods of the central point and  $T$  is the low threshold for Canny optimized algorithm. To penalize the local irregularity in curves, an energy term is added to Eq. (13):

$$\frac{\partial I_m}{\partial t} + F_c \cdot \frac{|\nabla(G * I_m)|}{|\nabla(G * I_m)|} + \mu \cdot g \cdot \delta_\varepsilon(I_m) = 0 \quad (15)$$

where  $g$  is an edge indicator function [11],  $\delta_\varepsilon(I_m)$  is the univariate Dirac function and  $\mu$  is an adjusting factor. In practice,  $g$  and  $\delta_\varepsilon(I_m)$  are defined as follows:

$$g = \frac{1}{1 + |\nabla G * I_m|^2} \quad (16)$$

$$\delta_\varepsilon(I_m) = \frac{1}{2\varepsilon} \cdot \left[ 1 + \cos\left(\frac{\pi \cdot I_m}{\varepsilon}\right) \right] \quad (17)$$

For all the experiments in this paper, both  $\mu$  and  $\varepsilon$  take the value of 1.5. This achieves a good performance both in the whole image for noise removal and in the local area for edge preservation.

### 3. EXPERIMENTAL RESULTS

In this section, testing experiments on synthetic images are presented to demonstrate the effectiveness of the proposed image restoration framework. All simulations are implemented in MATLAB.

There are four ideal scenes (images) containing four men with different postures, respectively, as shown in Figure 4(a). For each of them, 60 extremely low-resolution rectangular images are generated by shifting up along the horizontal, vertical and diagonal directions, *i.e.* 20 in each direction. Figure 4(b) gives the examples. Fig. 4(c) shows the preliminary results of linear interpolation by maximum likelihood estimation. After noise removal via the Canny operator based level set evolution, the final outputs are illustrated in Fig. 4(d). It can be seen that the men shapes are restored to a great extent compared with the measured images. It is sufficient to further image understanding.

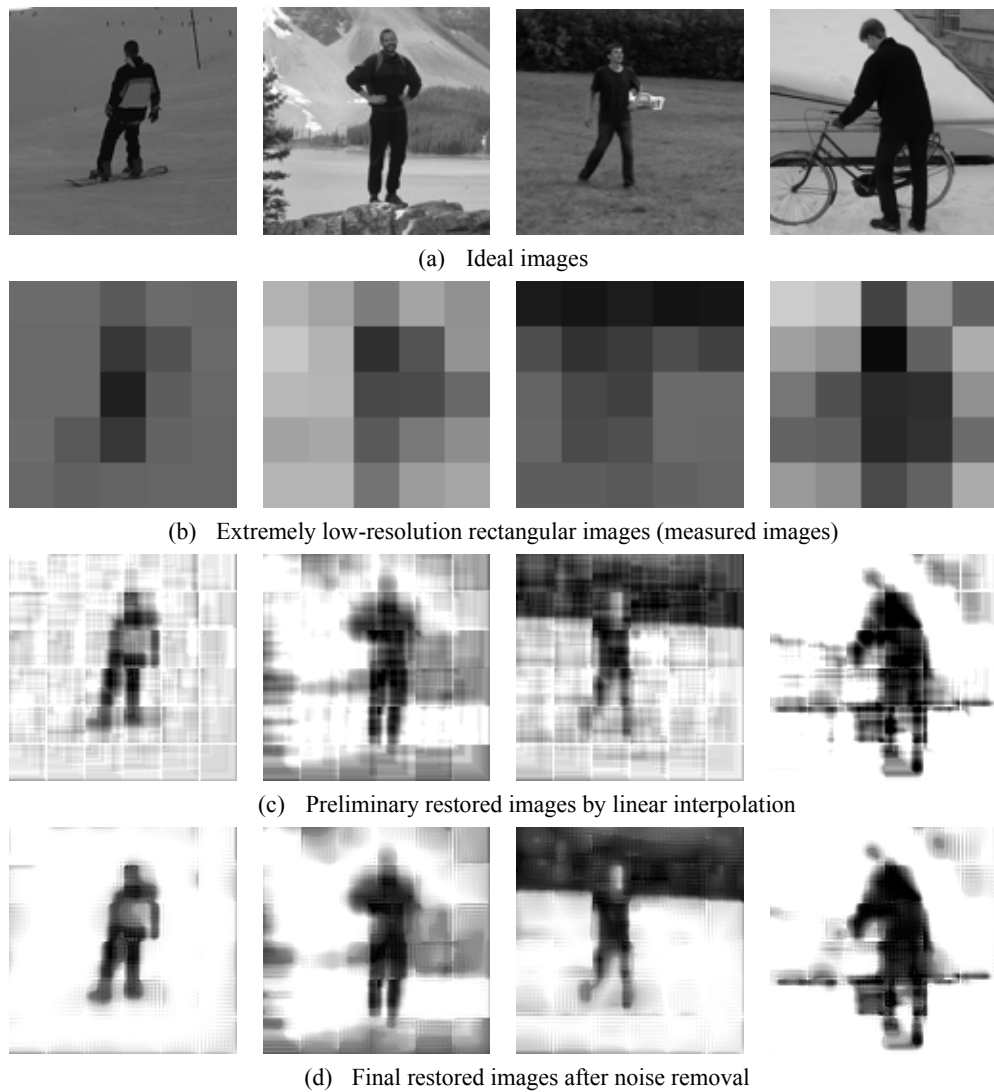


Figure 4: Results showing the proposed framework works to image restoration

## 4. CONCLUSIONS

In this paper, a unique image restoration framework is presented specifically for extremely low-resolution rectangular images, which ensures further application of the rectangular poor-pixels detector. To meet the demand of fast processing, only a few measured images shifted up to subpixel level along the horizontal, vertical and diagonal directions are needed to join the fusion operation. In order to still achieve a high-quality restored image, the two-stage image restoration strategy is then adopted. Based on maximum likelihood estimation, the preliminary restored image is linearly interpolated. By the modified level set evolution with Canny operator for noise removal, the final restored image is improved further. The experimental results have shown that the proposed framework is effectively performed in image restoration. It is a sensible step towards subsequent image understanding and object identification.

## REFERENCES

1. P. Wen and X. Wang, Multiprocessor poor pixels image acquisition system with low power consumption, *Chinese Journal of Scientific Instrument*, vol. 27, no. 6, suppl., pp. 1358–1359, 2006.
2. S. Farsiu, M. D. Robinson, M. Elad and P. Milanfar, Fast and robust multiframe super resolution, *IEEE Trans. Image Process.*, vol. 13, no. 10, pp. 1327–1344, 2004.
3. M. Elad and Y. Hel-Or, A fast super-resolution reconstruction algorithm for pure translational motion and common space-invariant blur, *IEEE Trans. Image Process.*, vol. 10, no. 8, pp. 1187–1193, 2001.
4. M. Elad and A. Feuer, Restoration of a single superresolution image from several blurred, noisy, and undersampled measured images, *IEEE Trans. Image Process.*, vol. 6, no. 120, pp. 1646–1658, 1997.
5. L. N. Trefethen and D. Bau, *Numerical Linear Algebra*, Philadelphia, USA: SIAM, 1997.
6. J. A. Sethian, *Level set methods and fast marching methods: evolving interfaces in computational geometry, fluid mechanics, computer vision, and materials science, 2nd ed.*, Cambridge University Press, 1999.
7. S. Osher and J. A. Sethian, Fronts propagating with curvature dependent speed: algorithms based on Hamilton-Jacobi formulations, *J Comput. Phys.*, vol. 79, no. 12, 1988.
8. D. Gil and P. Radeva, A regularized curvature flow designed for a selective shape restoration, *IEEE Trans. Image Process.*, vol. 13, no. 11, pp. 1444-1458, 2004.
9. C. Li, C.-Y. K, J. C. Gore and Z. Ding, Minimization of region-scalable fitting energy for image segmentation, *IEEE Trans. Image Process.*, vol. 17, no. 10, pp. 1940-1949, 2008.
10. J. Canny, A computational approach to edge detection, *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 8, no. 6, pp. 679-698, 1986.
11. C. Li, C. Xu, C. Gui and M. D. Fox, Level set evolution without re-initialization: a new variational formulation, in *Proc IEEE Conf Computer Vision and Pattern Recognition*, 2005.