# Modeling Image Motion in Airborne Three-Line-Array (TLA) Push-broom Cameras

Guimin Jia, Xiangjun Wang, Hong Wei, and Zhaocai Zhang

# Abstract

This paper presents an image motion model for airborne three-line-array (TLA) push-broom cameras. Both aircraft velocity and attitude instability are taken into account in modeling image motion. Effects of aircraft pitch, roll, and vaw on image motion are analyzed based on geometric relations in designated coordinate systems. The image motion is mathematically modeled by image motion velocity multiplied by exposure time. Quantitative analysis to image motion velocity is then conducted in simulation experiments. The results have shown that image motion caused by aircraft velocity is space invariant while image motion caused by aircraft attitude instability is more complicated. Pitch, roll, and yaw all contribute to image motion to different extents. Pitch dominates the along-track image motion and both roll and yaw greatly contribute to the cross-track image motion. These results provide a valuable base for image motion compensation to ensure high accuracy imagery in aerial photogrammetry.

# Introduction

Aerial photogrammetry is a geomatic method of collecting ground information by using photographic images captured by airborne devices, such as airplanes, helicopters, unmanned airborne vehicles (UAV) or other airborne vehicles (Baltsavias, 1999; Paine and Kiser, 2003; Petrie and Walker, 2007). A line scan camera is one of two principal digital cameras used in aerial photogrammetry, and the other is a frame camera. Generic frame cameras capture an object as a 2D image by using 2D array sensors; while a line scan camera captures an object as a line by using 1D sensors. To create a 2D image with a line scan camera, an object is captured line-by-line by either moving the camera over the object or moving the object under the camera. Compared to a frame camera, a line scan camera usually has higher resolution but low cost, its optics is also simple and compact, and image size is flexible in the along-track direction. A three-line-array (TLA) push-broom camera, which has three linear CCDs being placed in the optical focal plane, can provide 3D information of an object through its stereoscopic views. The TLA push-broom camera is capable of recovering exterior orientation parameters directly from images

Hong Wei is with the School of Systems Engineering, University of Reading, Whiteknights, Reading, RG6 6AY, United Kingdom. acquired by itself. It constructs along-track stereoscopy from the same platform with short time intervals so that the failure in image registration caused by changes of lighting condition is avoided to a great extent (Hofmann *et al.*, 1982; Hofmann, 1988; Petrie, 2005; Toutin, 2006; Chen *et al.*, 2007; Petrie and Walker, 2007; Wang *et al.*, 2009).

# A Brief History of TLA Cameras

The TLA principle was introduced and patented in the 1980s by the pioneer, Dr. Otto Hofmann (Hofmann et al., 1982; Hofmann, 1985; Hofmann, 1987; Hofmann, 1988). The German Modular Optoelectronic Multispectral Stereo Scanner (MOMS-02) was the first sensor system which used the TLA principle, and in 1993 it was flown on the Space Shuttle Mission STS55 and in 1996 it was installed in the PRIRODA-Module of the MIR Space Station (Fraser and Shao, 1996; Sandau, 2010). Both the High Resolution Stereoscopic (HRS) camera which was equipped on SPOT5 in 2002 and the stereo camera which was installed on Chang'E-1 in 2007 exploited the TLA principle (Toutin, 2006; Sun et al., 2008). With the development of spaceborne TLA cameras, the design of airborne TLA cameras was initiated. The first operational TLA airborne camera system was the Digital Photogrammetry Assembly (DPA). Comprehensive tests were performed by the Institute for Photogrammetry, University of Stuttgart, Germany since 1995. These tests delivered basic experience and knowledge to improve pilot systems to offer operational systems (Fritsch, 1997). The first commercially available digital airborne TLA camera system, ADS40 from Leica Geosystems, was introduced in the year 2000 at the ISPRS Congress in Amsterdam. Through the Position and Orientation System which was designed by Applanix Corporation of Canada, the position information from GPS and the attitude information from the inertial measurement unit (IMU) are combined to provide estimates of the aircraft's trajectory (Sandau, 2010). Indeed, a line scan camera needs to be operated on a very stable imaging platform that does not change its attitude and altitude during flight (Petrie, 2005). Otherwise gaps and double imaging will occur in consecutive strip images. When an airborne platform is subject to atmospheric turbulence, it may cause random and unpredictable changes in attitude and altitude with consequent effects on the imaging system (Sun, 2006). GPS/IMU systems can be used with airborne digital cameras to provide direct georeferencing or to assist in the triangulation process. As a result the accuracy of airborne surveying and mapping is improving.

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# **Research Motivation**

There are two conventional lens organizations in TLA cameras: (a) a single lens covering the three linear sensor arrays in its focal plane, and (b) three lenses, each for one linear sensor array, with converging optical axes. The former has advantage of simple mechanical design, while image distortion may be introduced due to edge effects of the lens in a wide-range projection. The latter may acquire high quality images, but its mechanical and optical systems are complex in order to make the three optical axes converge. In Chang'E-1 spacecraft mission, a new type of TLA camera was introduced, in which the three line arrays are selected from 2D CCD arrays using a single lens (Sun et al., 2008). This organization has advantages of compact structure, light weight, and easy assembly. However, it has a longer exposure time compared with purpose-built TLA CCD cameras. Especially when it is used in an airborne platform flying at a high speed (for example reconnaissance missions) with frequent attitude changes, image motion will be significant. This paper analyzes these scenarios with the focus on image motion modeling and simulation.

Image motion is defined as displacement of image points on CCDs caused by relative movement between a camera platform and objects to be captured during image exposure. Two error sources are identified: (a) aircraft velocity, and (b) aircraft attitude instability. This paper first explores the image motion caused by aircraft velocity. Due to angular motion during image exposure, the image motion caused by aircraft velocity is no longer along the flight direction. Second, the image motion caused by attitude instability is modeled by image motion velocity multiplied by exposure time. The image motion velocity is derived from aircraft's pitch, roll, and yaw separately. Finally, simulation experiments are conducted using the established image motion model with detailed analysis with respect to aircraft velocity and attitude instability. The paper ends with conclusions and future research.

# Three-line-array Push-broom Camera

As shown in Figure 1, photoelectric imaging elements of a TLA push-broom camera consist of three linear CCD arrays,

Fight Direction Rs Figure 1. Schematic diagram of three-line-array CCD camera. which are placed in the optical focal plane; l, n, and r stand for forward-looking, nadir-looking, and backward-looking, respectively. The three linear CCD arrays are parallel to each other, and perpendicular to the flight direction. During flight, l, n, and r scan the terrain simultaneously in the same scanning period from different perspectives, thereby produce three overlapping strip images (L<sub>s</sub>, N<sub>s</sub>, and R<sub>s</sub>) (Hofmann, 1988; Hofmann *et al.*, 1982).

It has been proven scientifically that the forwardlooking, nadir-looking, and backward-looking views provide sufficient information for photogrammetric reconstruction and stereoscopy is formed by using any pair from the three views (Gruen and Zhang, 2001 and 2002).

# Modeling Image Motion in Airborne TLA Push-broom Camera

# **Establishment of Coordinate Systems**

In this paper, four coordinate systems are defined for establishing the computational model of image motion (Refer to Figures 2, 3, 4, and 5 for symbols used in the coordinate systems.):

- 1. Local geographic coordinate system  $F_g(O_g X_g Y_g Z_g)$ At a point on the Earth's surface, its local geography is treated as flat ground.  $O_g$  is a random point on the Earth's surface, axis  $O_g X_g$  is parallel to the horizontal, and axis  $O_g Z_g$  vertically points to the center of the Earth.
- 2. Aircraft platform coordinate system  $F_b(O_b X_b Y_b Z_b)$ The aircraft platform coordinate system is fixed to the aircraft, and has its origin at the aircraft's center of gravity. Assuming that the camera is fixed on to the aircraft and the optical center of the lens is also at the center of the aircraft gravity. The camera coordinate system is the same as the aircraft platform coordinate system.



Figure 2. Schematic diagram of image motion caused by aircraft velocity.





The orientation of the three axes is defined as follows:

1. If the aircraft has a plane of symmetry, then  $X_b$  and  $Z_b$  lie in that plane.  $X_b$  is chosen to point forward stretching from the tail to the nose of the aircraft;



Figure 5. Schematic diagram of image motion caused by yaw.

- 2.  $Y_{b}$  is a horizontal axis stretching from one wing tip to the other; and
- 3.  $Z_b$  is chosen to point downward and always perpendicular to plane  $O_b X_b Y_b$ .

The Euler angles, known as yaw, pitch, and roll, describe the attitude and orientation of the aircraft. In this paper, roll is the rotation about  $O_b X_b$ , pitch refers to the rotation around  $O_b Y_b$ , and yaw refers to the rotation about  $O_b Z_b$ .  $\psi$ ,  $\theta$ , and  $\phi$  stand for yaw, pitch, and roll, respectively. The positive and negative attitude angles are defined as follows:

- 1. the clockwise refers to a positive angle when observed along the positive direction of a coordinate axis;
- 2. otherwise the attitude angle is negative.
- 3. The flight path coordinate system  $F_k(O_k X_k Y_k Z_k)$ .
- 4. The flight path coordinate system has its origin at the aircraft's center of gravity. The  $O_k X_k$  axis points to the direction of aircraft velocity. The  $O_k Z_k$  axis is in the symmetric plane of the aircraft and points to the Earth's surface.
- 5. Image plane coordinate system  $F_i(oxy)$ .

The image plane coordinate system is a 2D coordinate system, a projection of 3D objects. The origin o lies on the intersection point of the camera optical axis and the image plane, axis ox is parallel to axis  $O_b X_b$ , and axis oy is parallel to axis  $O_b Y_b$ . The distance between o and  $O_b$  is the focal length f.

Finally, it should be noted that all the coordinate systems are right-handed and orthogonal, and  $O_g X_g$  is set to be parallel to  $O_k X_k$  in order to simplify the computational model.

# **Image Motion Modeling**

For a camera mounted on a flying aircraft, there is always relative movement between the camera and observed objects during image exposure. When the aircraft flies stably, the attitude angles ( $\psi$ ,  $\theta$ ,  $\phi$ ) are steady and the aircraft velocity is the primary cause of image motion. When the attitude instability caused by air flows is taken into account, the uncertainty of attitude angles is added into image motion. Image motion caused by both aircraft velocity and attitude instability is analyzed in this section.

#### Image Motion Caused by Aircraft Velocity

Assuming the attitude angles are steady and set to be zero, the aircraft platform coordinate system  $F_b$  is the same as the flight path coordinate system  $F_k$ , and the aircraft velocity is along the  $O_b X_b$  direction. The image motion  $\delta$  caused by the aircraft velocity can be calculated by Equation 1 (Prinz, 1985):

$$\delta = f \frac{V_s}{H} \tau \tag{1}$$

where f is the focal length of the imaging system,  $V_s$  is the speed of flight, H is the distance between the target object and the CCD camera, and  $\tau$  is exposure time.

In practice, it is hard to keep the aircraft's attitude angles zero, and there always exist non-zero angles  $(\psi, \theta, \phi)$ during image exposure. Therefore Equation 1 is modified to meet the requirement. When the attitude angles are not zero,  $F_b$  is no longer the same as  $F_k$ . As shown in Figure 2, with attitude angles  $(\psi, \theta, \phi)$ , the nadir point shifts from O' to O''. The angular motion can be illustrated by coordinate transformations so that the aircraft velocity is transformed from  $F_k$  to  $F_b$ .

The image motion is mathematically modeled by image motion velocity multiplied by exposure time. The exposure time is constant so that image motion velocity  $\vec{\delta}_v$  is used to describe the change rate and direction of image motion. When the attitude angles are  $(\psi, \theta, \phi)$  during image exposure,  $\vec{\delta}_v$  is calculated based on rotating rules in coordinate systems:

$$\vec{\delta}_v = \delta/\tau = f \frac{\vec{V}}{|OO''|} \tag{2}$$

$$\vec{V} = R_{\psi\theta\phi} \ \vec{V}_s \tag{3}$$

$$\left|OO''\right| = H \frac{1}{\cos\theta\cos\phi} \tag{4}$$

where  $V_s = [V_s, 0, 0]^T$ ,  $R_{\psi\theta\phi}$  is a rotation matrix from the flight path coordinate system  $F_k$  to the aircraft platform coordinate system  $F_b$ .  $R_{\psi\theta\phi}$  can be expressed as:

 $R_{\psi\theta\phi} = R_{\phi} R_{\theta} R_{\psi}$ 

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} (5)$$

Substituting Equation 3, Equation 4, and Equation 5 in Equation 2, the image motion velocity caused by aircraft velocity is obtained as shown in Equation 6:

$$\vec{\delta}_{v} = [\delta_{vx}, \delta_{vy}]^{T} \\ = \left[ f \frac{V_{s}}{H} \cos\phi \, \cos^{2}\theta \, \cos\psi, f \frac{V_{s}}{H} \cos\phi \, \cos\theta \\ (\sin\phi \, \sin\theta \, \cos\psi - \cos\phi \, \sin\psi) \right]^{T}$$
(6)

#### Image Motion Caused by Aircraft Attitude Instability Image Motion Velocity Caused by Pitch

As shown in Figure 3,  $\alpha_1$  is the angle between the forward-looking and the nadir-looking,  $\alpha_2$  is the angle between the backward-looking and the nadir-looking.

Assuming that pitch  $\theta$  varies during exposure, and the angular drift rate of pitch is, it can be seen from Figure 3 that the image motion velocity in the across-track direction is zero. Image motion velocity on the three line arrays (*l*, *n*, and *r*) caused by pitch can be computed by Equation 7, Equation 8, and Equation 9, respectively.

$$\vec{\delta}_{\theta l} = [\delta_{\theta lx}, 0]^T = [-\frac{f}{\cos \alpha_1} \dot{\theta}, 0]^T$$
(7)

$$\vec{\delta}_{\theta n} = [\delta_{\theta nx}, 0]^T = [-f\dot{\theta}, 0]^T$$
(8)

$$\vec{\delta}_{\theta T} = [\delta_{\theta TX}, 0]^T = [-\frac{f}{\cos \alpha_2} \dot{\theta}, 0]^T$$
(9)

# Image Motion Velocity Caused by Roll

Figure 4 illustrates the aircraft roll. It rolls at an angular drift rate of  $\dot{\phi}$ , so that the direction of image motion velocity is in the across-track direction or along the *oy* axis. Image motion velocity on the three line arrays (*l*, *n*, and *r*) caused by roll can be computed by Equation 10, Equation 11, and Equation 12, respectively:

$$\vec{\delta}_{\phi l} = [0, \delta_{\phi l y}] = [0, \sqrt{f^2 + y_l^2 \dot{\phi}}]$$
(10)

$$\vec{\delta}_{\phi n} = [0, \delta_{\phi ny}] = [0, \sqrt{f^2 + y_n^2 \dot{\phi}}]$$
 (11)

$$\vec{\delta}_{\phi r} = [0, \delta_{\phi ry}] = [0, \sqrt{f^2 + y_r^2} \dot{\phi}]$$
 (12)

where  $y_l$  is the coordinate of the image point on the forward-looking CCD sensor,  $y_n$  is the coordinate of the image point on the nadir-looking CCD sensor, and  $y_r$  is the coordinate of the image point on the backward-looking CCD sensor.

#### Image Motion Velocity Caused by Yaw

As shown in Figure5, the aircraft yaws at an angular speed of  $\psi$ . For the forward-looking and backward-looking, the yaw causes motion in both the *ox* and *oy* directions. However, the image motion velocity on the nadir-looking CCD is only in the along-track direction or the *ox* direction. It is calculated by Equation 13:

$$\vec{\delta}_{\psi n} = [\delta_{\psi nx}, 0]^T = [-y_n \dot{\psi}, 0]^T$$
 (13)

The magnitude of forward-looking image motion velocity is calculated by Equation 14:

$$\left|\vec{\delta}_{\psi l}\right| = \left|\sqrt{(f \tan \alpha_1)^2 + y_l^2} \, \dot{\psi}\right| \tag{14}$$

where  $\alpha_1$  is the angle between forward-looking and nadir-looking CCD sensors.

The magnitude of backward-looking image motion velocity is calculated by Equation 15:

$$\left|\vec{\delta}_{\psi r}\right| = \left|\sqrt{(f \tan \alpha_2)^2 + y_r^2} \, \dot{\psi}\right| \tag{15}$$

where  $\alpha_2$  is the angle between backward-looking and nadir-looking CCD sensors.

Based on the analysis and calculation above, the image motion velocity caused by attitude instability in an airborne TLA push-broom camera is modeled as follows: 1. Forward-looking Sensor

$$\vec{\delta}_{for} = \begin{cases} x \ direction & -\frac{f}{\cos\alpha_1} \dot{\theta} - \sin\left(\arctan\frac{y_l}{f\tan\alpha_1}\right) \sqrt{(f\tan\alpha_1)^2 + y_l^2} \dot{\psi} \\ y \ direction & \sqrt{f^2 + y_l^2} \dot{\phi} + \cos\left(\arctan\frac{|y_l|}{f\tan\alpha_1}\right) \sqrt{(f\tan\alpha_1)^2 + y_l^2} \dot{\psi} \end{cases}$$
(16)

2. Nadir-looking Sensor

$$\vec{\delta}_{nadir} = \begin{cases} x \ direction \ -f\dot{\theta} - y_n \ \dot{\psi} \\ y \ direction \ \sqrt{f^2 + y_n^2} \ \dot{\phi} \end{cases}$$
(17)

3. Backward-looking Sensor

$$\vec{\delta}_{back} = \begin{cases} x \ direction & -\frac{f}{\cos\alpha_2} \ \dot{\theta} - \sin\left(\arctan\frac{y_r}{f\tan\alpha_2}\right) \sqrt{(f\tan\alpha_2)^2 + y_r^2} \ \dot{\psi} \\ y \ direction & \sqrt{f^2 + y_r^2} \ \dot{\phi} + \cos\left(\arctan\frac{|y_r|}{f\tan\alpha_2}\right) \sqrt{(f\tan\alpha_2)^2 + y_r^2} \ \dot{\psi} \end{cases}$$
(18)

# **Results and Discussion**

Equations 6 through 18, have shown that the image motion in an airborne TLA push-broom camera is affected by various factors, such as aircraft velocity, attitude angles, focal length of the imaging system, flight altitude, the angle between forward-looking and nadir-looking CCD sensors, the angle between backward-looking and nadir-looking CCD sensors, and the location of image point on CCD arrays.

# **Results of Image Motion Caused by Aircraft Velocity**

According to reported aerial TLA cameras (Neukum, 1999; Neukum and HRSC-Team, 2001; Tsuno *et al.*, 2004; Sun

et al., 2008; Green et al., 2011), parameters involved in this simulation are given as follows: f = 80 mm,  $V_s = 120 \text{ m/s}$ , H = 2500 m,  $\alpha_1 = \alpha_2 = 23.02^\circ$ ,  $\dot{\phi} = \dot{\theta} = \dot{\psi} = 0.01 \text{ rad/s}$ ,  $\tau = 20 \text{ ms}$ , the pixel size of CCD is 6.8  $\mu$ m, and the number of pixels on each line array is 10,000. According to Equation 6, the image motion velocity caused by aircraft velocity is  $\delta_v = [3.8324\mu\text{m/ms}, -0.1168\mu\text{m/ms}]^T$ , and the magnitude of image motion during exposure is 76.648  $\mu$ m in the alongtrack direction and 2.336  $\mu$ m in the across-track direction. The distribution of image motion velocity caused by aircraft velocity on the three CCD arrays is illustrated in Figure 6.



Figure 6. Image motion velocity on (a) forward-looking, (b) nadir-looking, and (c) backward-looking CCD sensors caused by aircraft velocity; the image motion velocity is displayed as arrows. The length of the arrow stands for image motion velocity magnitude, and the direction of the arrow for the motion direction; x and y are the coordinates on the image plane coordinate system.

Figure 6 demonstrates that image motion velocity caused by aircraft velocity is space invariant, and it has the same magnitude and direction on the forward-looking, nadir-looking and backward-looking sensors. With the calculated value from the model, the image motion can be compensated by techniques, such as optical methods, mechanical methods, and TDI (Time Delay and Integrate) method. Furthermore, algorithms used for space invariant image restoration can also be employed as a software method for compensation of this type of image motion.

#### **Results of Image Motion Caused by Attitude Instability**

Image motion caused by aircraft attitude instability is more complicated than that caused by aircraft velocity as shown in Equations 16 through 18. In this paper, the analysis is conducted to answer three questions: (a) which CCD array is affected mostly by attitude instability, (b) which attitude angle affects the image motion mostly, and (c) what are features of the image motion when changes of the three attitude angles take place at the same time?

# Image Motion Velocity Caused by Pitch, Roll, and Yaw Individually

This simulation intends to discover to what extent the image motion velocity will be introduced by changes of a single attitude angle, hence to indentify which CCD array is affected greatest by attitude instability.

Parameters are given as: f = 80 mm,  $\phi = \dot{\theta} = \dot{\psi} = 0.01$  rad/s,  $\alpha_1 = \alpha_2 = 23.02^{\circ}$ . There are 10,000 pixels on a line array sensor. The image motion can be calculated and modeled

according to Equations 7 through 15. The distribution of image motion velocity on the forward-looking, nadir-looking, and backward-looking CCDs caused by pitch, roll, and yaw is illustrated in Figures 7, 8, and 9, respectively.

Figure 7 demonstrates that the image motion velocity caused by pitch is space invariant on all the three line array sensors. It points to the along-track direction, and the magnitude is the same for each line array sensor. However the nadir-looking sensor has smaller magnitude than those of the other two line array sensors. In other words, the nadir-looking sensor is less affected by pitch than the forward-looking and backward-looking sensors.

Figure 8 shows that the image motion velocity caused by roll is distributed identically on the three line array sensors. At each point of a line array it is along the acrosstrack direction. Its magnitude is different, and related to image coordinate. The further from the center of the sensor, the greater is the magnitude.

Figure 9 demonstrates the image motion velocity caused by yaw. For the nadir-looking sensor, the image motion velocity is zero at the center of the sensor (y = 0). The further from the center, the greater is the magnitude. For the forward-looking and the backward-looking sensors, both magnitude and direction of the image motion velocity vary as a function of image pixel positions at the line array sensor. The further from the center of the sensor, the greater is the magnitude, and the more drifts the image motion velocity direction away from the cross-track direction. Compared to the nadir-looking sensor, the forward-looking and the backward-looking sensors are more seriously affected by aircraft yaw.



velocity is displayed as arrows. The length of the arrow stands for image motion velocity magnitude, and the direction of the arrow stands for the motion direction; x and y are the coordinates in the image plane coordinate system.



Table 1 shows the simulation results quantitatively. It indicates that the magnitude of image motion velocity caused by attitude instability is smaller for the nadir-looking sensor than for the forward-looking and backward-looking sensors. Among the three attitude angles, yaw has less impact on image motion velocity than pitch and roll.

# Image Motion Velocity on One Line Array Caused by Integrated Attitude Changes

This section applies pitch, roll, and yaw simultaneously to one line array sensor. It examines how the uncertainty affects the image motion in individual line array sensors. Assuming that the drifting rate of attitude angles varies from 0.000001 rad/s to 0.05 rad/s, the image motion velocity against it is calculated by Equations 7 through 15 and the results are illustrated in Figure 10.

In the simulation,  $y_l$ ,  $y_n$ , and  $y_b$  in Equations 10 through 15 are given as 34 mm (0.5 \* 6.8  $\mu$ m \* 10,000

pixels, i.e., one-half of the length of pixel array). Figure 10 demonstrates that the image motion velocity magnitude caused by pitch and roll is greater than that caused by yaw. Quantitatively, for the forward-looking sensor, when the angular drifting rate is of 0.01 rad/s, the image motion velocity magnitude caused by yaw is about 0.43 mm/s and that caused by pitch and roll is about 0.85 mm/s. When the angular drifting rate is greater than 0.01 rad/s, the image motion velocity magnitude increases rapidly with increase of the angular drifting rate. For the nadir-looking sensor, at the same angular drifting rate of 0.01 rad/s, the image motion velocity magnitude caused by yaw is about 0.35 mm/s and that caused by pitch and roll is about 0.8 mm/s. Again the image motion magnitude increases rapidly with increase of the angular drifting rate when the angular drifting rate is greater than 0.01 rad/s. For the backward-looking sensor, the trend of image motion velocity magnitude is similar to that of the forward-looking sensor.

TABLE 1.	MAGNITUDE OF IMAGE	MOTION VELOCITY OF	TLA PUSH-BROOM	CAMERA	CAUSED BY	ATTITUDE IN	NSTABILITY
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	pitch		roll		yaw	
	Min	Max	Min	Max	Min	Max
	[µm/ms]	[µm/ms]	[µm/ms]	[µm/ms]	[µm/ms]	[µm/ms]
Forward-looking	0.869	0.869	0.8	0.869	0.34	$0.481 \\ 0.34 \\ 0.481$
Nadir-looking	0.8	0.8	0.8	0.869	0	
Backward-looking	0.869	0.869	0.8	0.869	0.34	



From the analysis above, it can be concluded that roll and pitch affect the image motion more than yaw.

# Analysis of Image Motion Velocity Caused by Attitude Instability through 3D View

3D views are applied to demonstrate features of the image motion velocity for pitch against yaw, and roll against yaw. This answers the question of which variables  $(\theta, \phi, \psi, y_l, y_n, \psi)$  $y_r$ ) dominate the magnitude and direction of image motion velocity. Taking the forward-looking sensor for example, features of the image motion velocity caused by attitude instability are illustrated in Figures 11 and 12 for motions in the x axis and y axis, respectively. In Figures 11 and 12, the size of symbols represents magnitude. Figure 11 shows that the image motion velocity direction of the forward-looking sensor in the along-track direction is mainly dominated by pitch. The image motion velocity magnitude is affected by  $y_l$ ,  $\theta$ , and  $\psi$ , among which  $\theta$  is the major factor. However, the space variance is mainly caused by  $y_l$  and  $\psi$ . This makes image motion compensation more difficult (Ayasso and Mohammad-Djafari, 2010; Tai et al., 2010).

Figure 12 shows that the image motion velocity direction of the forward-looking sensor in the across-track direction is mainly dominated by roll and yaw. The image motion velocity magnitude is affected by  $y_l$ ,  $\phi$ , and  $\psi$ , among which  $\phi$  and  $\psi$  contribute more to the magnitude. The magnitude distribution is also space variant. When roll and yaw change in the same direction (both clockwise or both anticlockwise), the image motion velocity magnitude becomes greater.

# Conclusions

In this paper, a computational model is established for airborne TLA push-broom cameras with regard to image motion caused by both aircraft velocity and attitude instability. Aircraft attitude angles (i.e., pitch, roll, and yaw) have been built into the image motion model through four coordinate systems, namely local geographic coordinate system, aircraft platform coordinate system, flight path coordinate system, and image plane coordinate system. The image motion is mathematically modeled in such a way that quantitative analysis can be conducted with given parameters under derived equations. The following conclusions can be made from experimental results based on the computational model of image motion:

- Image motion caused by aircraft velocity is space invariant; both the magnitude and direction of the motion velocity during exposure time are uniform over the image plane regardless image pixel positions.
- Aircraft velocity has the same impact to the forward-looking, nadir looking, and backward-looking sensors with regard to image motion.
- Aircraft pitch (rotation around the  $y_b$  axis in the aircraft platform coordinate system) during image exposure introduces greater image motion to the forward-looking and backward-looking sensors than to the nadir-looking sensor. The introduced image motion is also space invariant.
- Aircraft roll (rotation around the  $x_b$  axis in the aircraft platform coordinate system) during image exposure has the same impact on the three sensors. However the motion magnitude varies along the y direction.





- Aircraft yaw (rotation around the z<sub>b</sub> axis in the aircraft platform coordinate system) during image exposure presents more complicated effects. Both magnitude and direction change with image pixel positions on the image plane. The model provides the detailed equations with which the magnitude and direction of image motion can be calculated at a fixed pixel position. The values can be used for image motion compensation to provide more accurate imagery.
- Among the three variables of pitch, roll, and yaw, yaw has the least effect to image motion. With increase of the attitude angle drifting rate, the image motion velocity is increased, but only after a certain point (0.01 rad/s in the experiment), this trend becomes significant.
- In the *x* direction on the image plane, the aircraft pitch dominates the direction of image motion velocity while both pitch and yaw decide the magnitude.
- In the y direction on the image plane, both roll and yaw influence the direction and magnitude of image motion. When roll and yaw change in the same direction (both clockwise or both anticlockwise), the image motion velocity magnitude becomes greater.

The above results can be used in compensation of image motion errors caused by both the aircraft velocity and attitude instability. Quantitative control can be applied to each variable under the image motion model. This is a part of the future work in the project. The work contributes to accurate image registration, stereo matching, and ground point positioning, hence to improve accuracy of aerial photogrammetry by using airborne TLA pushbroom cameras.

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